Online Bin Packing with Predictions

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Work with Shahin Kamali (York) and Kimia Shadkami (Manitoba)

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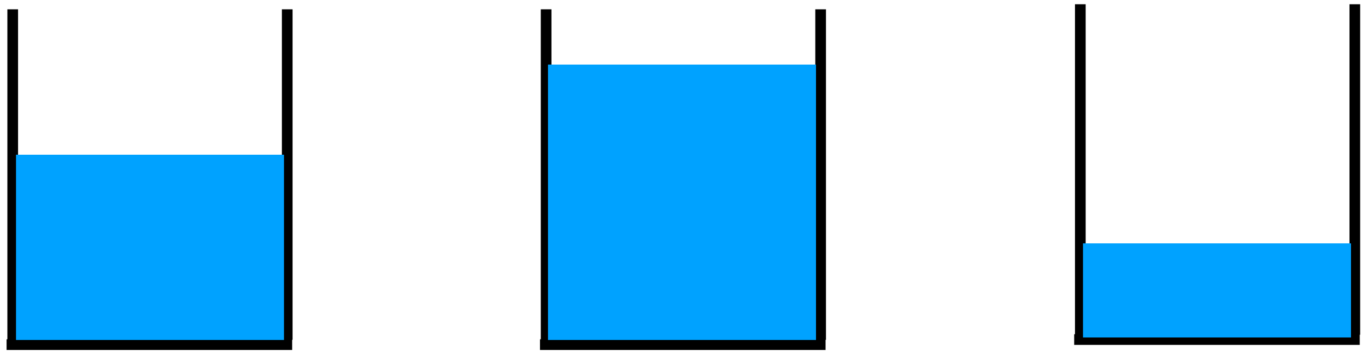

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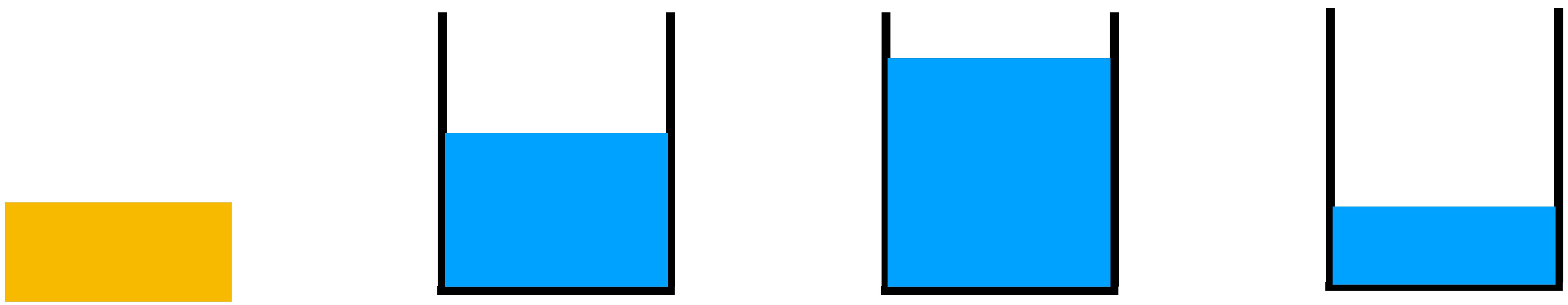

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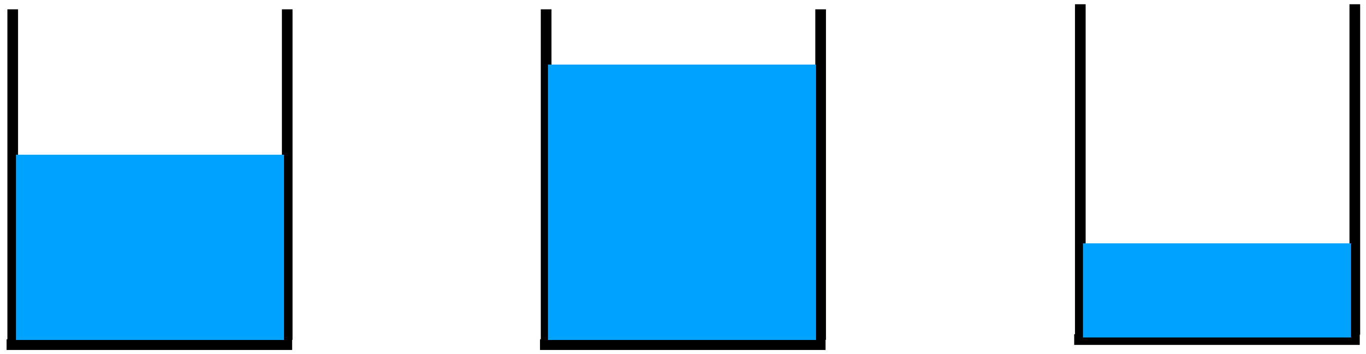

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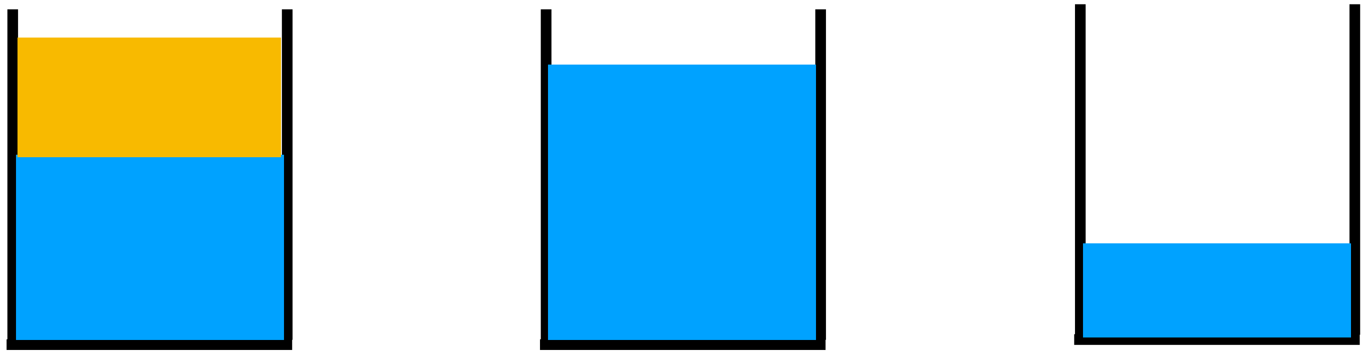

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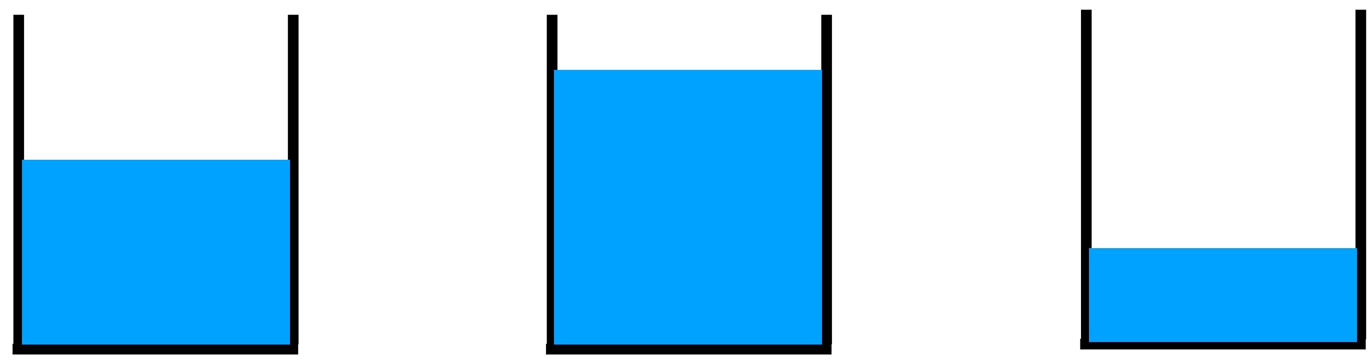

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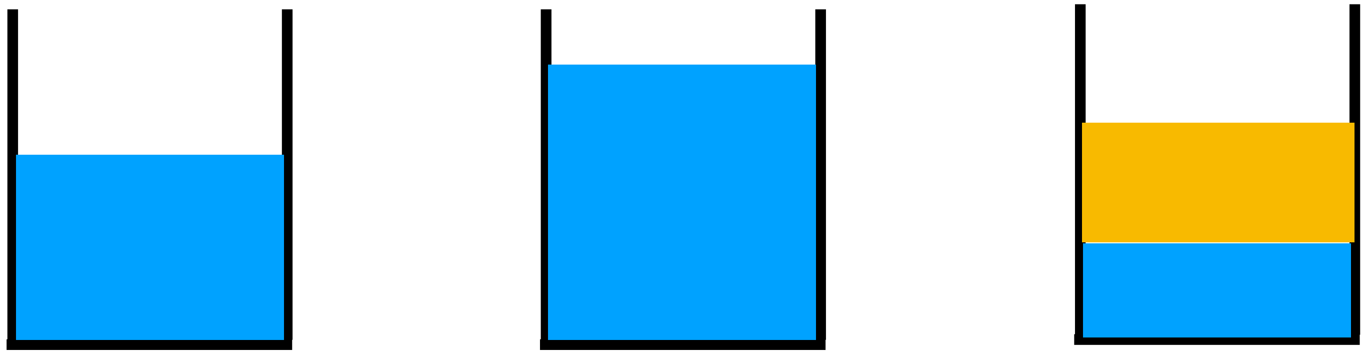

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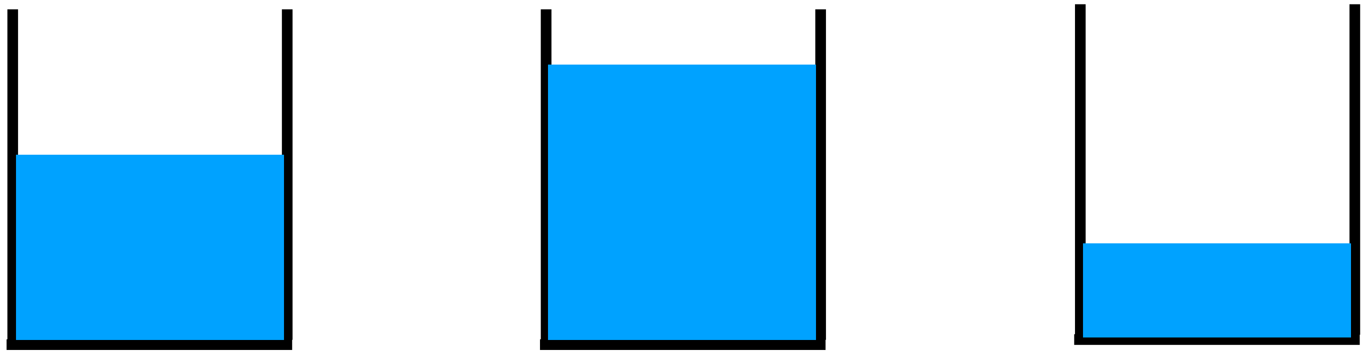

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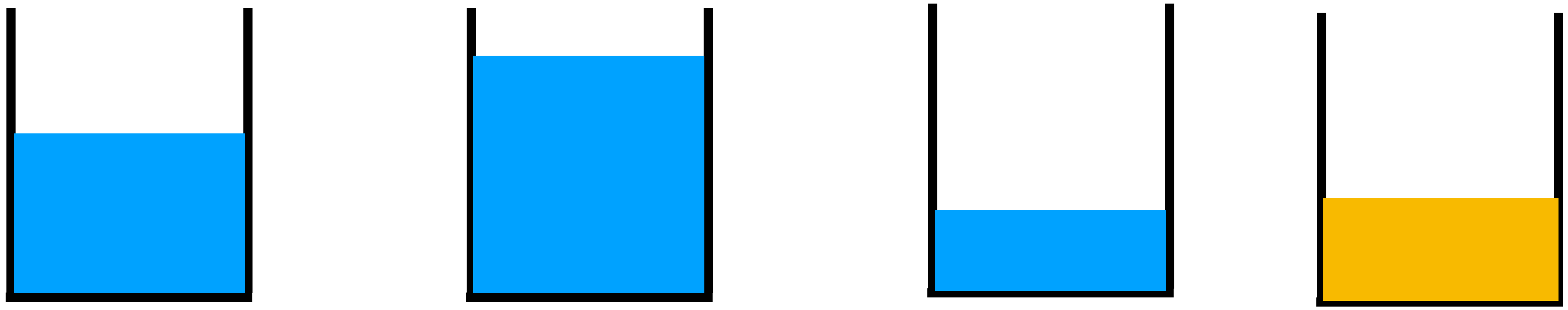

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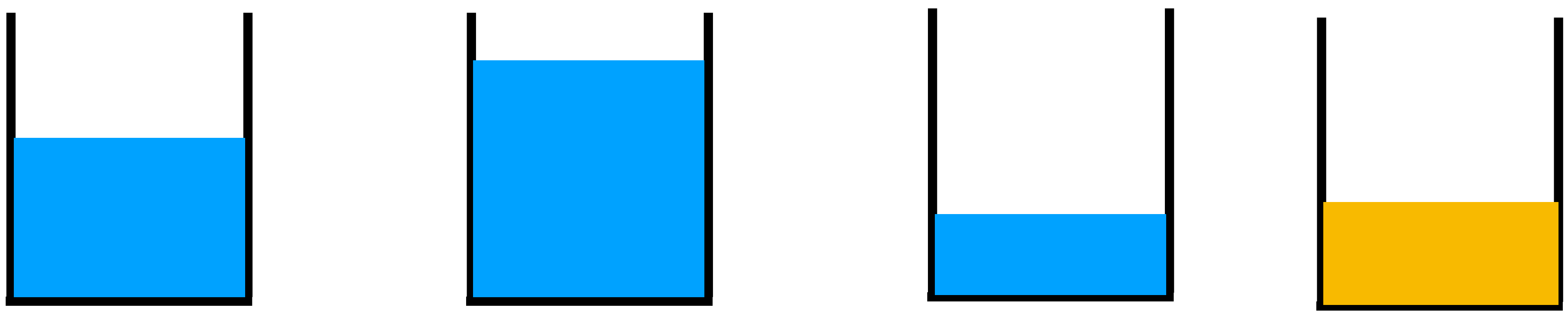

Objective: Minimize the (asymptotic) competitive ratio

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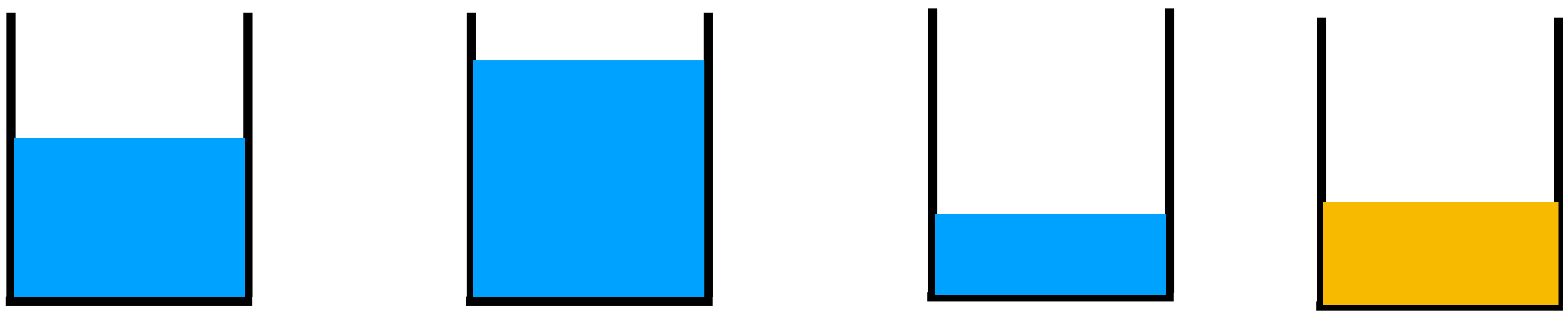

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Many applications (e.g., cloud computing)

Some important results

Best known **upper** bound : 1.57829 [Balogh, Békési, Dósa, Epstein, Levin 2018]

Best known **lower** bound : 1.54278 [Balogh, Békési, Dósa, Epstein, Levin 2021]

FIRST-FIT, BEST-FIT have competitive ratio 1.7 [Johnson et al. 1974]

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Enhance the standard model of so as to leverage some additional information about the input

Online algorithms with predictions

[Lykouris and Vassilvitskii 2018]

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competitive ratio with error η

Predictions that are "learnable" (e.g., sampling of the input)

Algorithms that degrade "gently" with error

Theoretical and experimental results

Side note: advice complexity of bin packing

Competitive ratio of algorithms with access to bits of “additional information”

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- **Trusted advice :** [Boyar, Kamali, Larsen and López-Ortiz 2016]

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  Trade-offs between advice size and competitive ratio, for **error-free** advice

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  Consistency-robustness tradeoffs for advice of a given size

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We assume a *discrete* model: The bin capacity is a constant k, and each item has integral size in $[1,k]$

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Frequency predictions are PAC-learnable

Profile of a bin packing sequence

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Fix a (large) constant M. We call the multiset that consists of $\lceil f_{x,\sigma} \cdot M \rceil$ items of size x

the **profile** of σ

We can compute the optimal packing of this profile set in $O(1)$ time

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Profile consists of 10 items of size 1, 3 items of size 2 and 2 items of size 3

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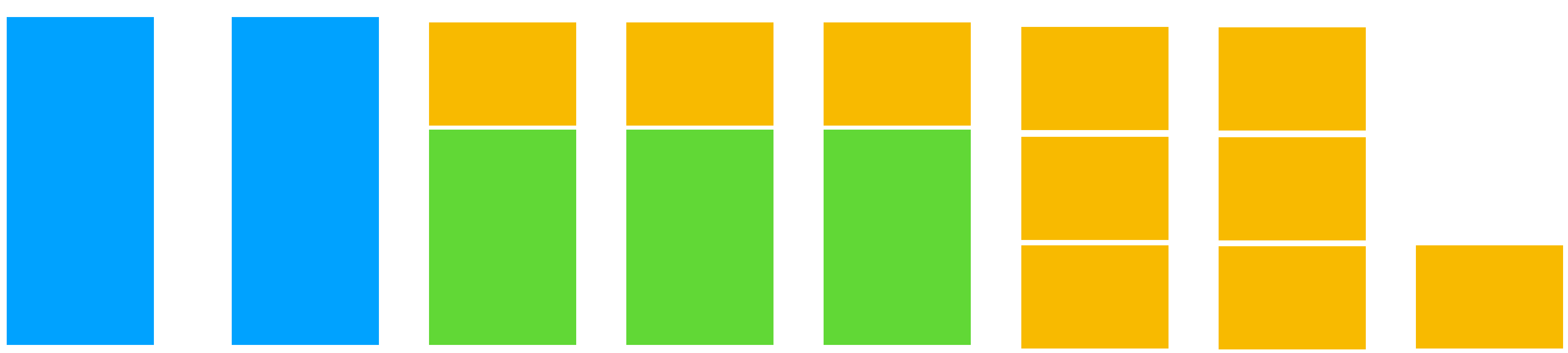

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Main idea: Pack the items by placing them in their profile placeholder

Algorithm opens profiles instead of single bins (virtually)

Items that were not predicted to appear are packed separately using FIRST-FIT

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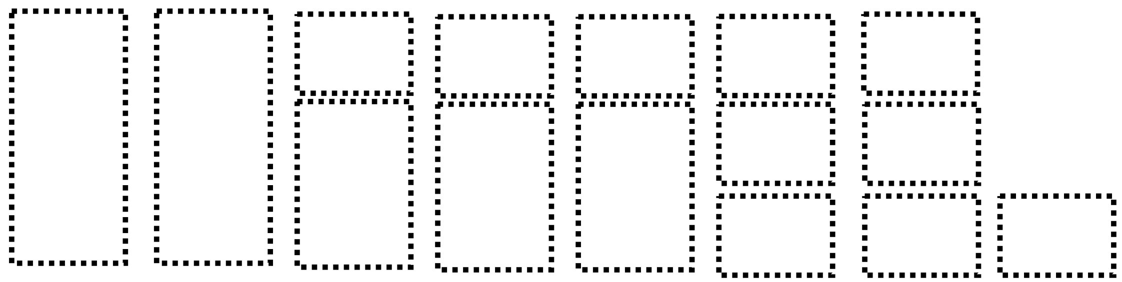

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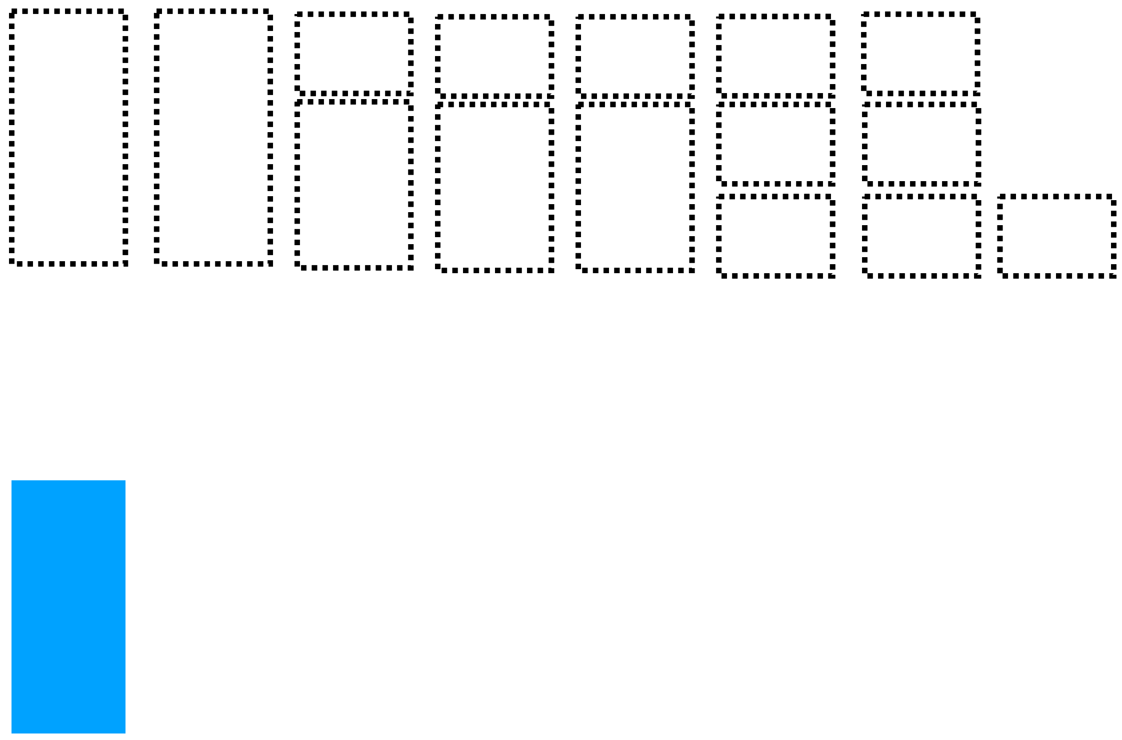

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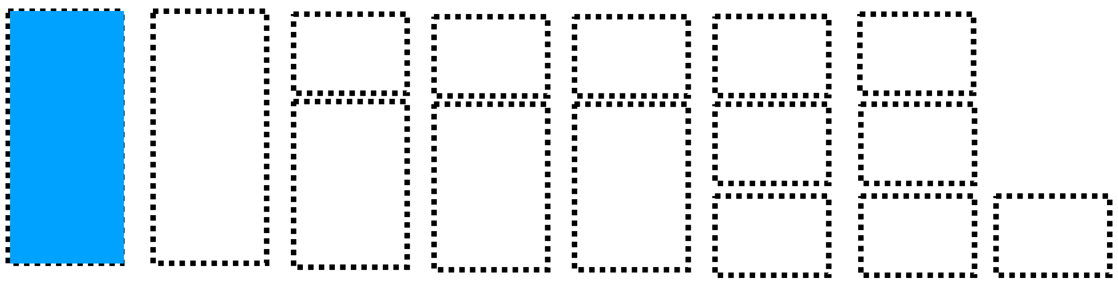

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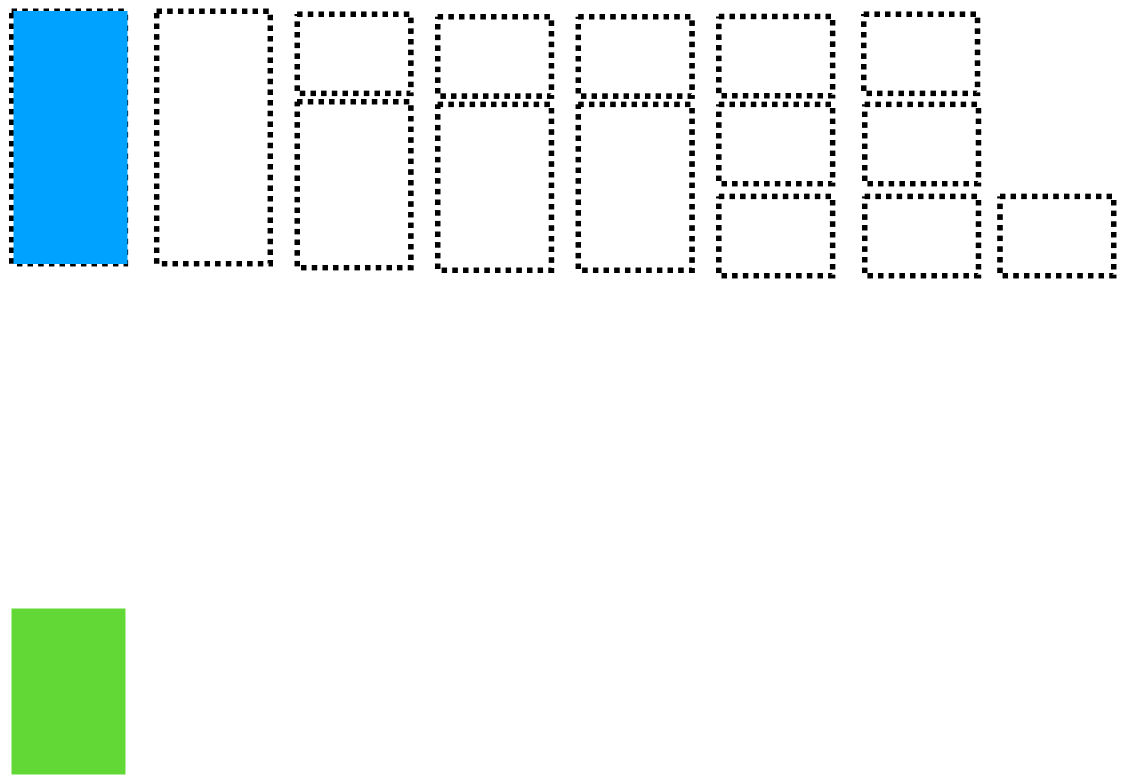

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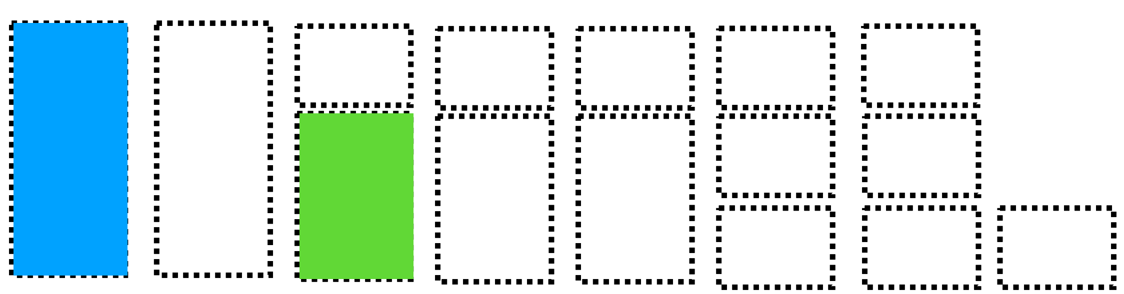

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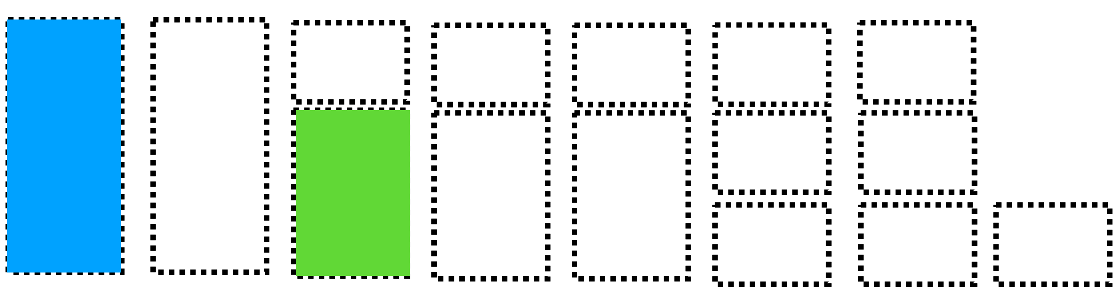

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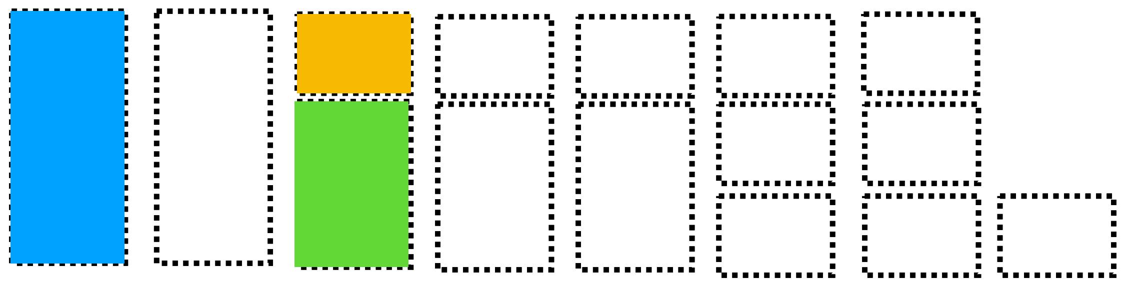

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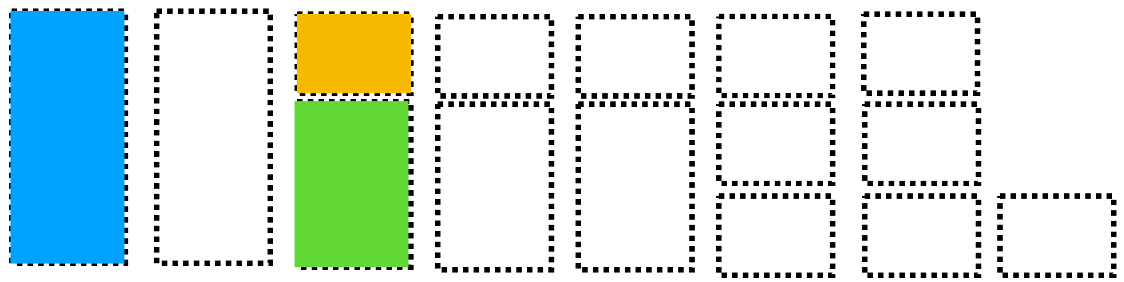

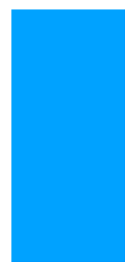

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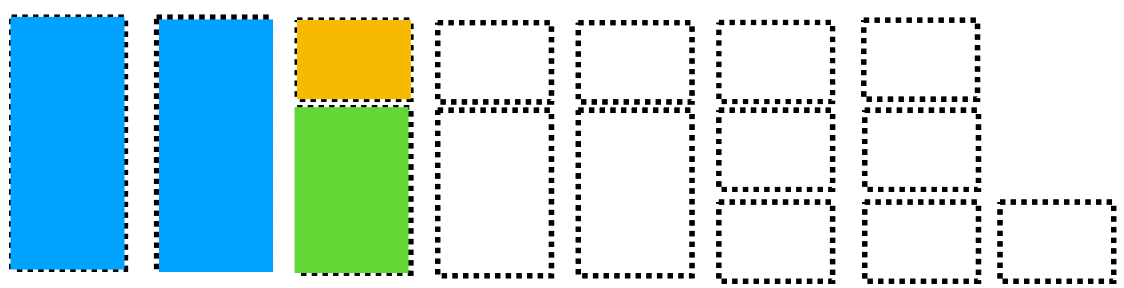

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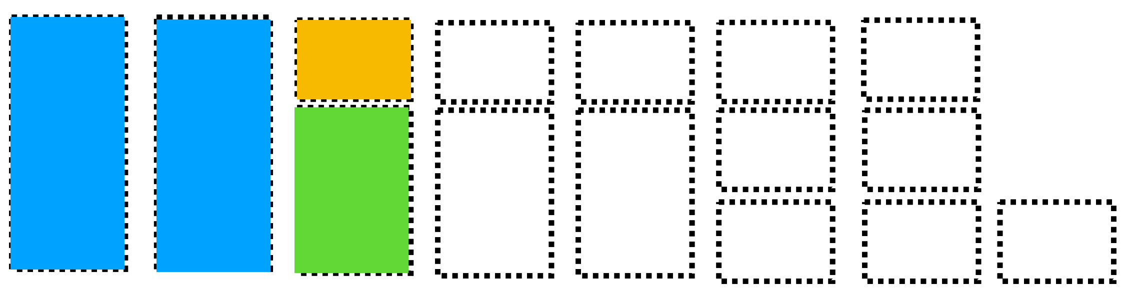

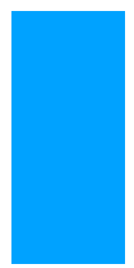

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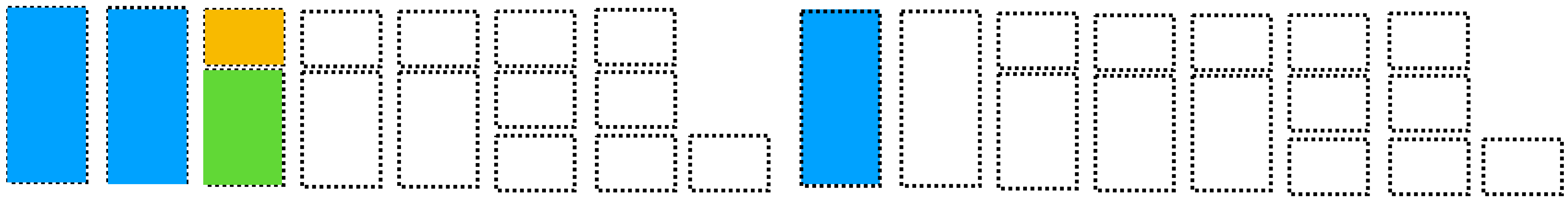

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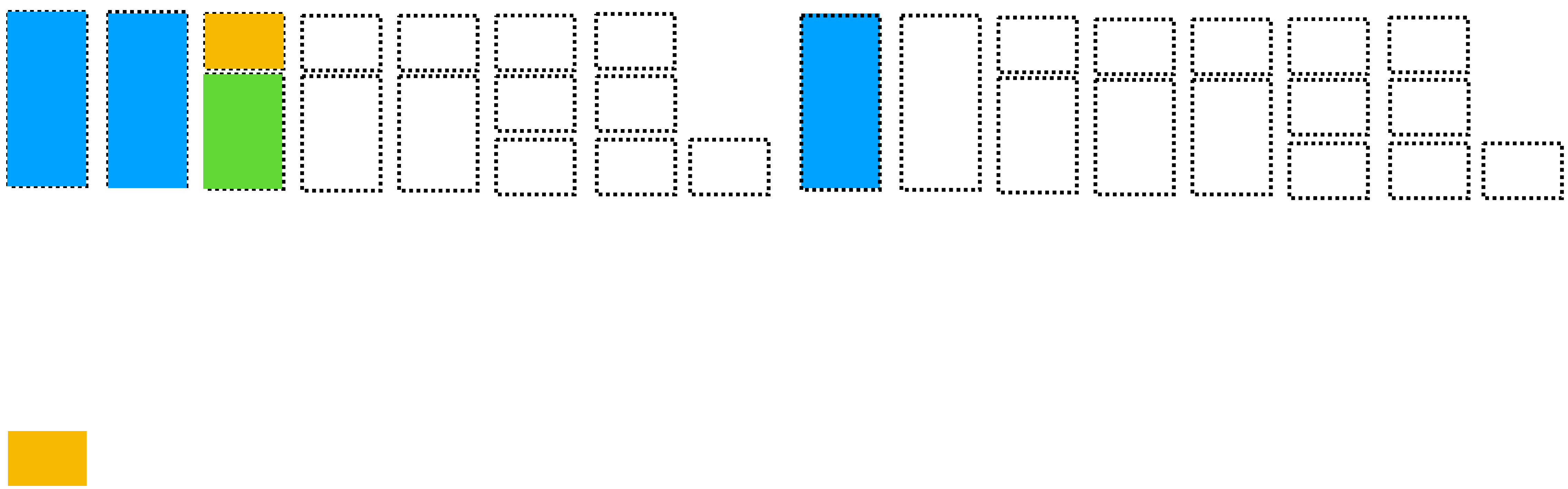

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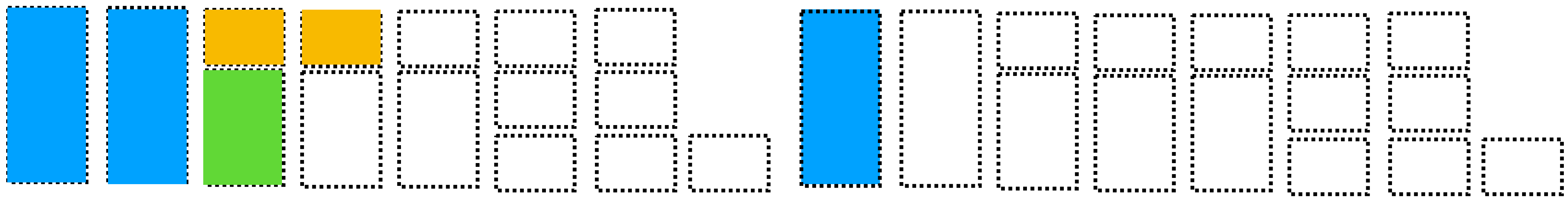

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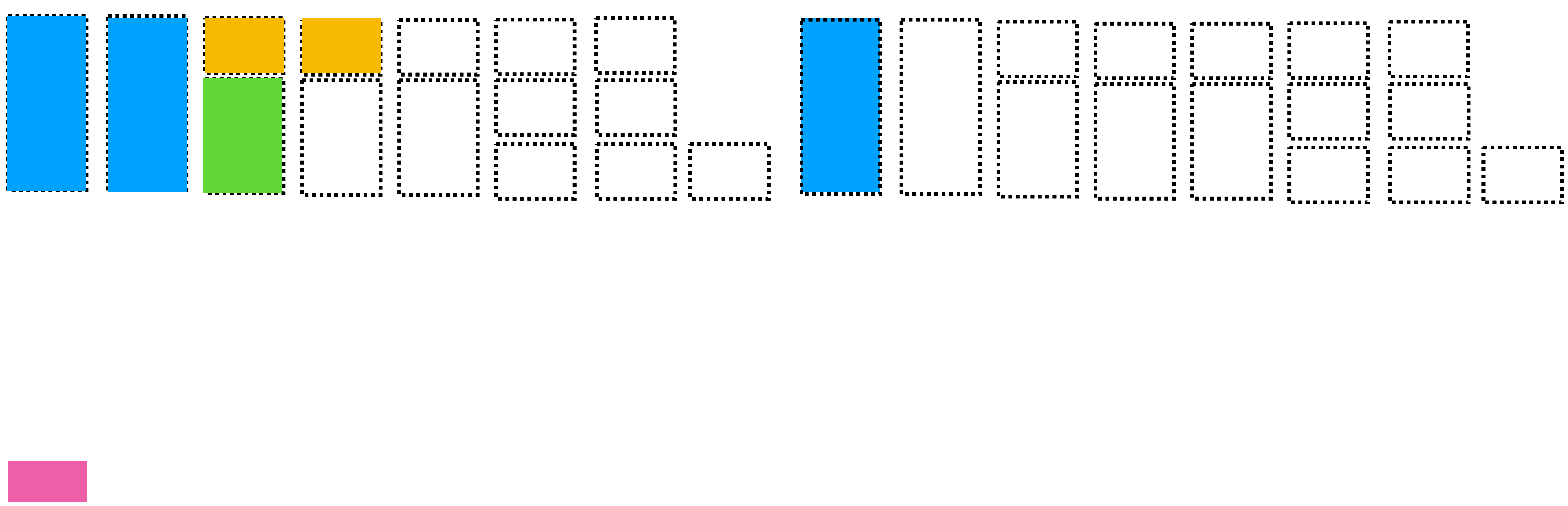

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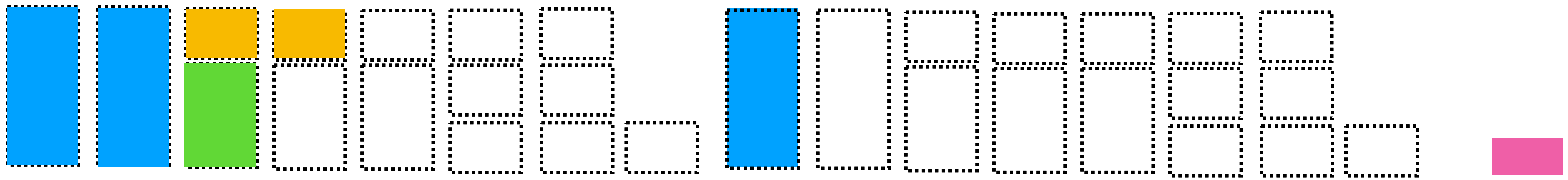

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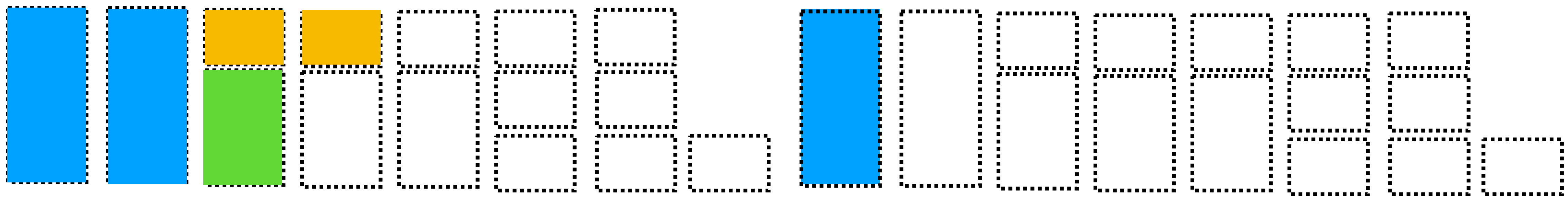

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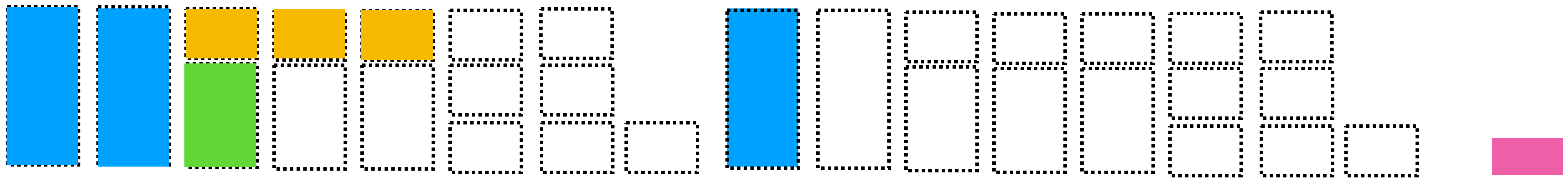

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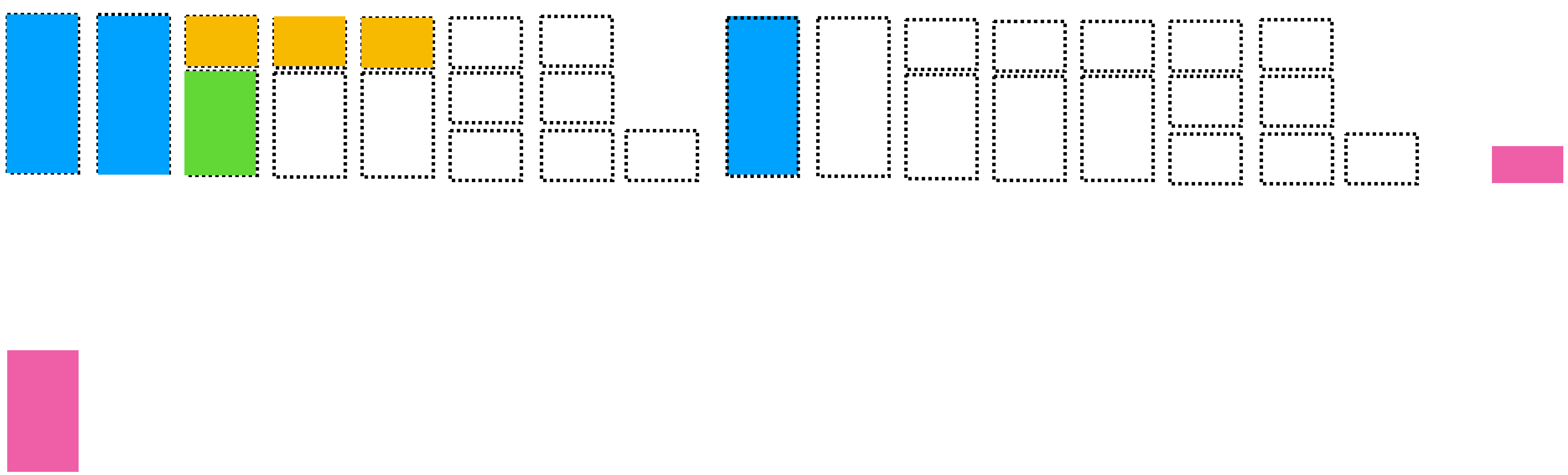

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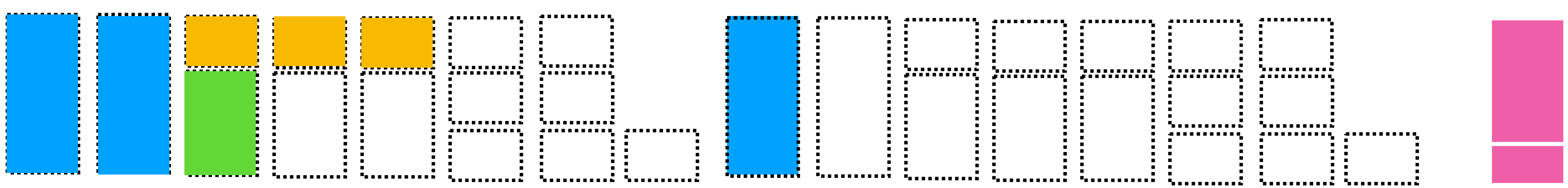

Some main results

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Theorem: Profile Packing has consistency $1 + \epsilon$, and competitive ratio at most

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Lower bound: Let $c < 1$ be a constant. For any $\alpha \leq c/k$, any algorithm that is $(1 + \alpha)$-consistent must have robustness at least $(1 - c)k/2$

Robustification

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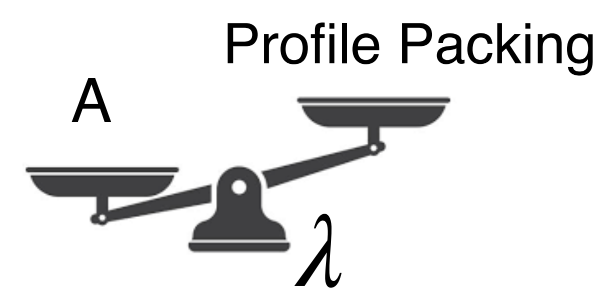
We define an algorithm **HYBRID**(λ)**,** where $\lambda \in [0,1]$ is a parameter chosen by the user

Main idea: Some items are served using a competitive algorithm (A), and others using Profile Packing



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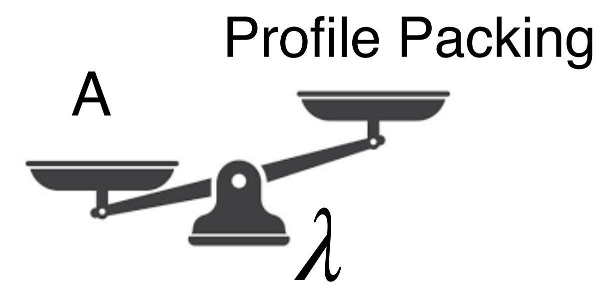

Algorithm :

— Keep track of the number of items served by PP, and the total number of items

Specifically: counters $ppcount(x)$ and $count(x)$ for all $x \in [1,k]$

— Upon arrival of an item of size x

- If there is an available placeholder, use it, and declare it a PP-item
- Otherwise, if $ppcount(x) \leq \lambda \cdot count(x)$, serve it using PP

  Else serve it using A

Robustification (results)

Theorem: HYBRID(λ) has competitive ratio at most

$$(1+\epsilon)((1+(2+5\epsilon)\eta k)\lambda + c_A(1-\lambda))$$, where $c_A =$ comp. ratio of A

Better results can be achieved assuming knowing some upper bound on the error

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Note: Some approaches that do not work:

— Skip the first step of the algorithm

— Algorithms along the lines of [Mahdian, Nazerzadeh and Saberi 2012]

Extensions

- Improvements when few items can be packed in a bin (VM placement)
- Sampling-based randomized online algorithms
- Handling fractional items

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If a fractional item appears, serve it separately using First Fit

We need a measure of the “integrality" of a sequence, e.g., $d(\sigma) = \sum_{x \in \sigma} |x - \lfloor x \rceil|$

This measure is too restrictive: no online algorithm with frequency predictions

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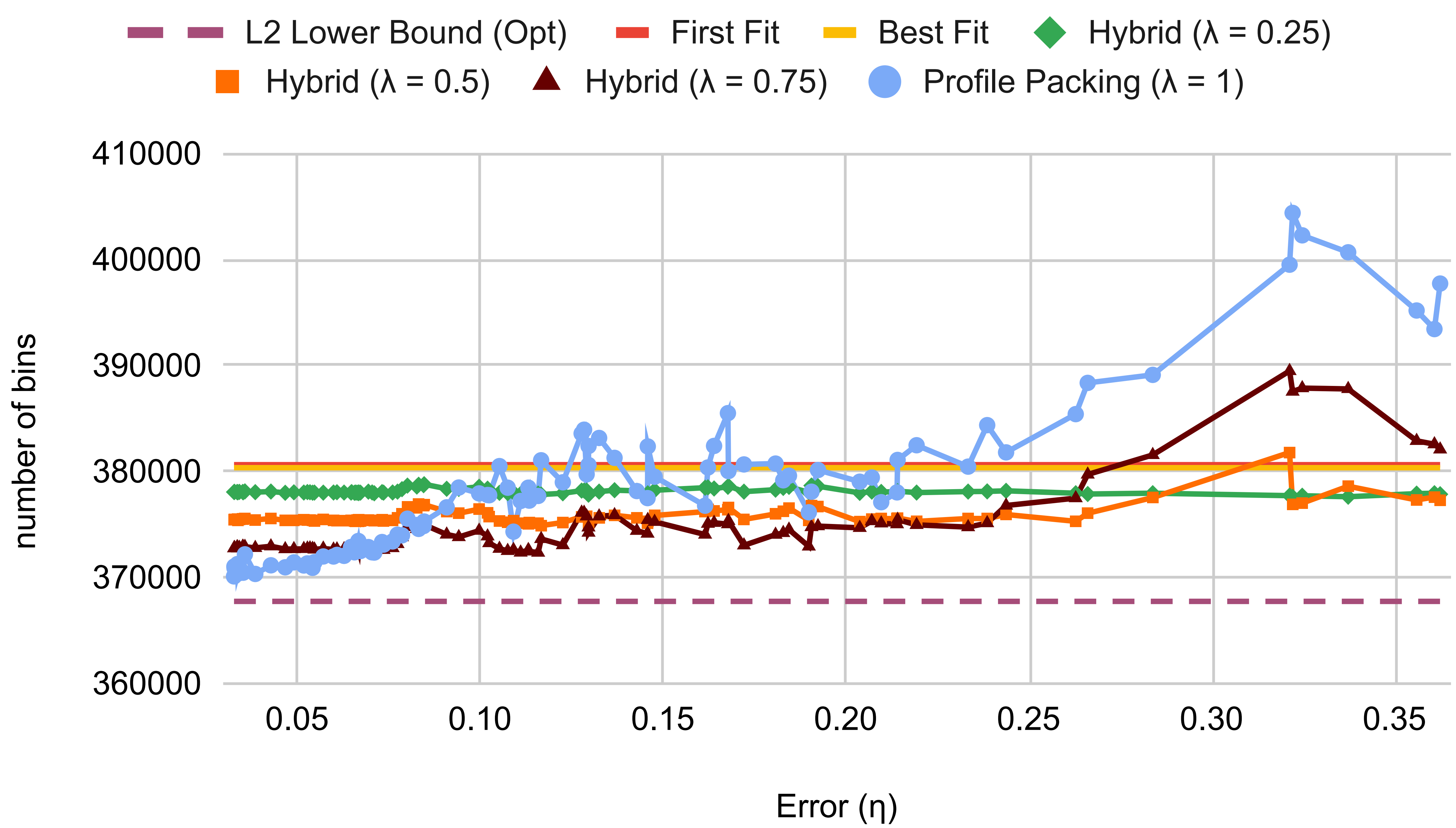
Alternative: $\hat{d}(\sigma) = \frac{\sum_{x \in \sigma, x \neq \lfloor x \rceil} x}{\sum_{x \in \sigma} x}$ (ratio of “fractional” sizes over total sizes)

Result: If an algorithm with frequency predictions has competitive ratio c for integral sizes, then we can transform it to an algorithm of competitive ratio $c + 2\hat{d}(\sigma)$ for fractional sizes

Experimental analysis

Weibull Fixed



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Thank you!

A sampling-based online algorithm

We can use the PAC-learnability of frequency predictions to obtain a randomized algorithm that mixes a robust algorithm A and Profile Packing

Result: For any $\epsilon > 0$, there is a randomized algorithm with s samples that has expected competitive ratio $(1-\delta)(1+\epsilon)((1+(2+5\epsilon)\eta k+\epsilon)+c_A\delta$, where $\delta = 1/\sqrt{2^{s\eta^2-k}}$